IPP-QM-3: Decoherence

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MT25

The course

1. Basic quantum formalism
2. Density operators and entanglement
3. Decoherence
4. The measurement problem
5. Dynamical collapse theories
6. Bohmian mechanics
7. Everettian structure
8. Everettian probability
9. EPR and Bell's theorem
10. The Bell-CHSH inequalities and possible responses
11. Contextuality
12. The PBR theorem
13. Quantum logic
14. QBism
15. Pragmatism and relational quantum mechanics
16. Wavefunction realism

Today

Decoherence introduced

Decoherence formalised

Consistent histories

Decoherence and branching

Phase space functions

Further applications of decoherence

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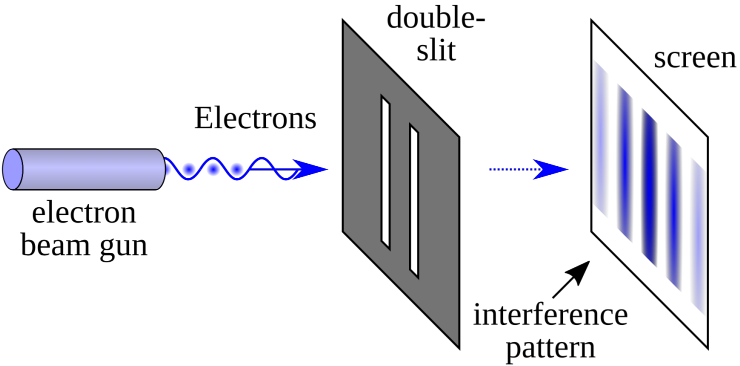
- Decoherence is ubiquitous in quantum mechanics and (all parties agree) of great foundational importance!
- Today, I'll introduce some of the details of decoherence.

Warmup: the double slit experiment



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- One might naïvely try to calculate them by summing over the probabilities of detection at the slits multiplied by the probabilities for detection at the screen conditional on detection at the slits.
- But in general in quantum mechanics there is an additional so-called interference term in the correct expression for the probability, and this term depends on both the wave components that pass through the slits.

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- Quanitatively, the density distribution of particles on the screen $\varrho(x)$ is given by (see e.g. Schlosshauer 2007, ch. 2):

$$\begin{aligned}\varrho(x) &= \frac{1}{2}\left|\psi_1(x) + \psi_2(x)\right|^2 \\ &= \frac{1}{2}\left|\psi_1(x)\right|^2 + \frac{1}{2}\left|\psi_2(x)\right|^2 + \mathrm{Re}\left\{\psi_1(x)\psi_2^*(x)\right\}\end{aligned}$$

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- (For a nice discussion, see (Maudlin 2019, p. 58).)

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- More recent surveys are given in Zeh (2003a), Zurek (2003), and in the books by Giulini et al. (1996, second edition Joos et al. 2003), and by Schlosshauer (2007).

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- Its best-known property is the suppression of coherence (i.e., quantum mechanical interference effects) in superpositions of states for the system (in a particular basis picked out by the subsystem-environment interaction).

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- In other words, let the first system be in state

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- Suppose that the Hamiltonian of the system contains some interaction term $\hat{H}_{\text{int}} = V(\hat{X} - \hat{x})$, where $\hat{X}$ and $\hat{x}$ are the position operators of the first and second particles, respectively.

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- We might even say that the second particle has 'measured' the position of the first.

Quantifying the entanglement

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- If we idealise it as having exactly two possible position states, $|\psi_{q_1}\rangle$ and $|\psi_{q_2}\rangle$, then the first particle's density operator evolves as

$$\rho_0 = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} \longrightarrow \rho_+ = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \langle\phi_2^+|\phi_1^+\rangle \\ \alpha^*\beta \langle\phi_1^+|\phi_2^+\rangle & |\beta|^2 \end{pmatrix}.$$

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  - when they have magnitude $|\alpha^*\beta|$, the first particle is in a pure state and so not at all entangled with the second particle;
  - if they are equal to zero, then the entanglement is maximal, and the quantum measurement algorithm gives the same predictions as it would were the first particle's state to be in a probabilistic mixture of the two positions.

Important point of clarification

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- This, to repeat, is really decoherence in a nutshell.

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- On the other hand, if the scattering is strong then $\langle \psi_2^+ | \psi_1^+ \rangle \approx 0$, and the entanglement is almost maximal.

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- It can be shown (Joos et al. 2003, pp. 63–7) that the rate is approximately given by

$$\langle x_1|\,\rho(t)\,|x_2\rangle = \langle x_1|\,\rho(0)\,|x_2\rangle \exp\left[-\Lambda t(x_1 - x_2)^2\right],$$

where $\Lambda \sim k^2 F\sigma/\lambda^2$, where F is the incoming particle flux, σ is the interaction cross-section, and λ is the wavelength.

Some decoherence timescales

Environment	Dust grain	Large molecule
CMB	1	10^{24}
Photons at room temp.	10^{-18}	10^{6}
Best laboratory vacuum	10^{-14}	10^{-2}
Air at normal pressure	10^{-31}	10^{-19}

Estimates of decoherence timescales (in seconds) for the suppression of spatial interferences over a distance Δx equal to the size a of the object. (From Schlosshauer 2007, p. 135)

Decoherence is fast!

Needless to say, the shortness of these timescales is truly astonishing and indicates the extreme speed and efficiency of decoherence. Our estimates demonstrate that spatial interference effects are extremely difficult to observe for "ordinary" objects (such as dust grains) immersed into similarly "ordinary" environments (such as thermal photons). (Schlosshauer 2007, pp. 134–5)

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- For the harmonic oscillator, one should think of the environment ‘measuring’ approximate eigenstates of position, or rather approximate joint eigenstates of position and momentum, so-called ‘coherent states’.
- It can be helpful to think in terms of ‘inertial frames’ (recall IPP-SR): the decoherence basis is like an ‘inertial frame in Hilbert space’, in which the description of the subsystem simplifies maximally. (I’ll return to this in a moment.)

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- Of course, absent some non-unitary dynamical process of a kind for which we have no evidence, a cat-plus-environment system remains in a superposition of live-cat and dead-cat states, even after decoherence!
- Decoherence gives us only *improper* mixtures, not *proper* mixtures!
- So decoherence alone does not solve the foundational problems of quantum mechanics! (Contrary to what one sometimes hears people saying!)

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- Although all these examples involve an *external* environment, there’s no need to make this distinction.
- There is, in fact, every reason to think that the microscopic degrees of freedom of even an isolated system suffice to destroy coherence between macroscopic superpositions of that system’s macroscopic degrees of freedom.

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Introducing consistent histories

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- For that, it's helpful to turn to the 'consistent histories' formalism developed by Gell-Mann & Hartle (1990).

The consistent histories formalism

- In this formalism, a *history* H is written as a time-ordered sequence of projectors:

$$H = \left\{ \hat{P}_1(t_1), \hat{P}_2(t_2), \ldots, \hat{P}_n(t_n) \right\},$$

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- Sometimes, $D(H, H') \approx 0$ is referred to as *medium decoherence*.

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- Then, the probability of a history H is $p(H) = D(H, H)$.
- In the absence of decoherence, one cannot assign probabilities to histories due to quantum interference.
- With sufficient coarse-graining, one can find a set of consistent histories; this is understood to be part of the story as to how classical physics emerges from quantum physics.

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- This means the condition does not uniquely select which set of histories is the 'correct' or 'physically relevant' one.
    - Example: one consistent set might describe a particle as having a definite position at different times; another equally valid consistent set might describe it as having a definite momentum at those times.

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- For any given quantum system, there isn't just one decoherent set of histories—rather, there are many (often incompatible) consistent sets which all satisfy the medium decoherence condition.
- This means the condition does not uniquely select which set of histories is the 'correct' or 'physically relevant' one.
  - Example: one consistent set might describe a particle as having a definite position at different times; another equally valid consistent set might describe it as having a definite momentum at those times.
- If the formalism permits many different 'storylines' for the same system, all equally valid but mutually incompatible, then what does it mean to say it 'describes reality'?

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- If the formalism permits many different 'storylines' for the same system, all equally valid but mutually incompatible, then what does it mean to say it 'describes reality'?
- Consistent histories then runs the risk of being a 'many-worlds without worlds' framework: a proliferation of incompatible but equally legitimate descriptions.

Responses to the worry

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1. Pragmatic responses:
   - Not all consistent sets are physically useful.
   - The ones that matter are those corresponding to quasiclassical variables (like macroscopic position, momentum, pointer states).
2. Gell-Mann & Hartle's 'quasiclassical realms' (1990):
   - Gell-Mann & Harte suggest that we focus on 'quasiclassical realms' that are stable under coarse-graining, robust to environmental decoherence, and approximately obey classical laws.
   - That narrows the field drastically: while many sets are mathematically consistent, only a few are dynamically stable and relevant to observers.

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But let’s return to the analogy with frames of reference in spacetime physics: there might well be a panoply of ‘legitimate’ frames of reference, but some are still better-adapted to the dynamics than others.

(Cf. Fletcher & Weatherall (2023ab) versus Linnemann *et al.* (2024) and Gomes (2025).)

Today

Decoherence introduced

Decoherence formalised

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Decoherence and branching

Phase space functions

Further applications of decoherence

The branching quantum state

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> *The importance of decoherence is: when it occurs, quantum-mechanical systems (approximately) develop a particularly natural branching structure. For decoherence is a process which constantly and (on sub-Poincaré-recurrent timescales) irreversibly entangles the environment with the system so as to suppress interference between terms of the decoherence-preferred basis. (We might say that the environment constantly measures the system and records the result.) If we idealize the dynamics as discrete, then at each branching event, the environment permanently records the pre-branching state, so that at each time the universal state is a superposition of states each of which encodes a complete record of where 'its weight' comes from.* (Wallace 2012, p. 88)

Decoherence and branching

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1. Deliberate human experiments: Schrödinger's cat, the two-slit experiment, Geiger counters, etc.
2. 'Natural quantum measurements', e.g. when radiation causes cell mutation.
3. 'Classically chaotic' processes: i.e., processes governed by Hamiltonians whose classical analogues are chaotic.

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2. 'Natural quantum measurements', e.g. when radiation causes cell mutation.
3. 'Classically chaotic' processes: i.e., processes governed by Hamiltonians whose classical analogues are chaotic.

The first is a relatively recent and rare phenomenon, but the other two are ubiquitous.

Branch counting and its discontents

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- There is no ‘finest’ choice of branching structure.
- As we fine-grain our decoherent history space, we will eventually reach a point where interference between branches ceases to be negligible, but there is no precise point where this occurs.
- As such, the question ‘How many branches are there?’ does not, by wide (but not universal! See Lecture 8), make sense.

Summary from Wallace

Decoherence causes the Universe to develop an emergent branching structure. The existence of this branching is a robust (albeit emergent) feature of reality; so is the mod-squared amplitude for any macroscopically described history. But there is no non-arbitrary decomposition of macroscopically-described histories into 'finest-grained' histories, and no non-arbitrary way of counting those histories. (Wallace 2012, pp. 101–2)

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- Given the (pure-state or mixed-state) position-space density matrix $\rho(x, x') \equiv \langle x| \hat{\rho} |x'\rangle$ of the system, the Wigner function on phase space is defined as

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}y e^{ipy} \rho(x + y/2, x - y/2),$$

where p is the conjugate momentum.

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  with $\int \mathrm{d}x \int \mathrm{d}p W(x,p) = \mathrm{Tr}(\hat{\rho}) = 1$.
- However, the Wigner function will in general take on *negative* values in some regions, so it cannot represent a proper probability distribution!

The Wigner function and decoherence

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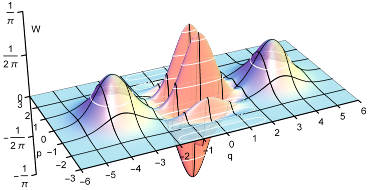

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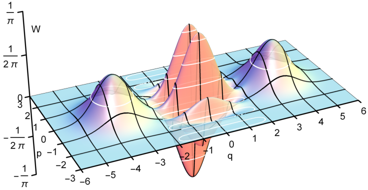

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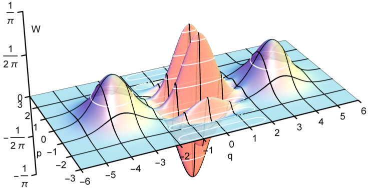

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- The main peaks, sometimes called the *direct peaks* are located in the classically-expected phase-space regions.

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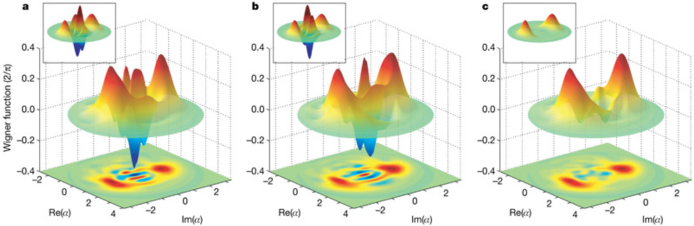
- As the system interacts with its environment, these are suppressed. E.g.:



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Other phase space functions

- There are other interesting phase space functions, e.g. the Husimi function and Q-function.
- Friederich (2024) has proposed an interpretation of quantum mechanics based upon the Q-function which (he claims) solves many of the interpretative problems of quantum mechanics!
- I'm somewhat sceptical (how did the improper mixtures turn into proper mixtures? See Wallace on the measurement problem in the next lecture), but this proposal is so recent that it has yet to receive serious engagement.

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- Another claim about the significance of decoherence relates to time asymmetry. Insofar as apparent collapse (branching) is indeed a time-directed process, decoherence will have direct relevance to the emergence of this 'quantum mechanical arrow of time'.
- Finally, it has been suggested that decoherence should be a useful ingredient in a theory of quantum gravity, as discussed e.g. by Kiefer (1994).

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- Next time: *the measurement problem*.

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